# Conceptions of "Problem-Solving", and Objectives of Mathematics Education: A Note

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#### Abstract

The conception of "problem-solving" argued these days by mathematics educationists in the United States is comparatively considered with the corresponding conceptions of Dewey and of progressivists being enlightened. Then "problem-solving" is reconsidered in the context of "objectives of mathematics education".

#### I. Introduction

"Problem-solving" is one of the most salient topics discussed by mathematics educationists these days in the United States. The National Council of Teachers of Mathematics (NCTM) made clear its view on the theme in "An Agenda for Action — Recommendations for School Mathematics of the 1980s" (abreviated "Agenda"), firstly recommending "problem-solving" by saying: "Problem solving must be the focus of school mathematics in the 1980s." ([N,p.2]) 'Problem-solving' as a currently advocated doctrine of mathematics education, which the author words with single inverted commas in order to make it understood that the word does not mean the performance of "solving (specific) problems", is, however, a new and old conception and one might say that recent discussion is another revival of the old one (cf. [KIJ3]). Indeed, as far as only the level of philosophy or, more precisely, of ideology is concerned, it is hard to find anything new or, at least, any progress in the recent arguments on 'problem-solving'. On the contrary, deplorably, does appear a sign of another popularization of the old philosophies and principles which were once to some extent highly qualified.

Considering the lack of progress of the old doctrine on 'problem-solving', we cannot help thinking that the recent movement of American mathematics educationists to problems approach is another reaction to the usually followed method of education, that is, the subject-oriented method. In fact, it could not be denied that one of the moments of current reflection on the subject-mastery method is the failure of the New Math. Thus, as far as it is viewed in this scheme of confrontation, recent fever of some American mathematics educationists over

'problem-solving' is a mere fashion that should be placed on the same level with that New Math.

Every revival of an old doctrine of education generally goes with a popularization of its fundamental ideology once highly qualified. If we are unmindful of such tendency, the transition of doctrines of education would be necessarily a cycle of temporal fashions. A method to cut this vicious cycle of fashions and to push up the arguments about principles of education to the level of science is to decide upon the conflicting ideologies and introduce some metatheory in which they should be adequately placed and related to each other. On the contrary, this cycle lasts as far as be repeated naive "well-intentioned" attempts to find anything good from every conceptions and methodologies that found on different ideologies and philosophies. Therefore, for mathematics pedagogists facing to the movement to 'problem-solving' today, the most expected way of study on the doctrine of 'problem-solving' should be to locate in a metatheoretical context the 'problem-solving' as a possible ideology, but not to treat it as a premised principle or as a field to be found and picked some "good".

The following consideration is made from such a point of view. Precisely, the author comparatively objectifies the conception of 'problem-solving' of today, enlightening Dewey's and the progressivists' conceptions about problems approach. And objectives of mathematics education will be considered with the doctrine of 'problem-solving' being referred to.

## II. Dewey's view on problem method

### 2-1. Dewey's methodology

When we make Dewey's educational theory and practice an issue, it is important to mind that the theory is coherent with his epistemological philosophy. As is generally known, he stood on a sort of empiricism. In fact, opposing to the traditional dogma, he advocated the conception of 'social inquiry'. As H. L. Friess pointed, Dewey thought: "philosophy's task, in this situation, is to further social inquiry into social processes and relations in order to reconstruct them and the beliefs that go with them" ([F, p.112]). Thus, the concept of 'social inquiry' turns out to be 'experimentalism'.

Dewey founded the Laboratory School at the University of Chicago for a practice of his philosophy of experimentalism ("Dewey School", 1896–1903). The school was, therefore, by intention an experimental school and not for a practice of some instructional methodology ([MaE, p.464]). Instructional methodology should be, on the contrary, the very thing that Dewey intended to decide following the experiments at the school.

## 2-2. Dewey's estimation for problem method

The views of criticism toward traditional method of education are different between Dewey and progressivists. Progressivists mainly opposed to the subject-oriented way of instruction on the ground of the relativism respecting the subject-matter and the principle of "clear thinking for the democratic living". But the reason why Dewey opposed to the traditional method is that he saw the intellectual poverty in the course of study according to the method.

K. C. Mayhew and A. C. Edwards, who were both teachers in Dewey School, stated:

"Custom and convention conceal from most of us the extreme intellectual poverty of the traditional course of study, as well as its lack of intellectual organization." ([MaE, p.468]). And Dewey stated:

".... the best that education can do during these years is to arouse intellectual interests which carry over and onwards." ([Dw2, pp.37, 38]).

Dewey, from such a point of view, conceived education in terms of experience. He stated: "It is a cardinal precept of the newer school of education that the beginning of instruction shall be made with the experience learners already have; that this experience and the capacities that have been developed during its course provide the starting point for all further learning." ([Dw3, p.74])

There was conceptioned the "principle of continuity of educational experience". The most essential of the conception is an "orderly development toward expansion and organization of subject-matter through growth of experience" ([Dw3, p.74]). And the development supposed here is exactly the one which we can see in those studies in "laboratories and institutes of research" ([Dw3, p.80]).

Dewey stated:

"That the conditions found in present experience should be used as sourses of problems is a characteristic which differentiates education based upon experience from traditional education. For in the latter, problems were set from outside." ([Dw3, p.79]).

Thus, the education conceived by Dewey turned out to be that of problem method in the original meaning, that is, of situation method. Here problems are regarded as stimuli to thinking. Dewey decided the following to be "part of the educator's responsibility"; that is, "to see equally to two things: First the problems grows out of the conditions of the experience being had in the present, and that it is within the range of the capacity of students; and, secondly, that it is such that it arouses in the learner an active quest for information and for production of new ideas" ([Dw3, p.79]).

In [Dw2], too, Dewey expressed, with some restriction, his esteem for project (problem,

situation) method as a possible alternative to traditional method in which, as is mentioned above, he saw the intellectual poverty:

"I do not urge it [project method] as the sole way out of educational confusion, not even in the elementary school, though I think experimentation with it is desirable in college and secondary school." ([Dw2, p.36])

Dewey regarded segregation of subjects as one of those which ruin interests ([Dw2, p.38]). Thus, reorganization of subject-matter was made an issue by him. Dewey stated:

"A reorganization of subject-matter which takes account of out-leadings into the wide world of nature and man, of knowledge and of social interests and uses, cannot fail save in the most callons and intellectually obdurate to awaken some permanent interest and curiosity." ([Dw2, p.38])

Problem method in the same sense as situation method necessarily realizes such a reorganization of subject-matter because, in this method, "material is drawn from any field as it is needed to carry on an intellectual enterprises ([Dw2, p.32])".

Needless to say, the usually followed segregation of subject-matter is a cultural product of the history of man that is based on some utilities and efficiencies. Therefore the dissolution of these categories is all nonsense. And we should not suppose that Dewey conceived the reorganization as the realization of alternative categorization. In fact, Dewey only pointed that "the organized subject-matter of the adult and the specialist cannot provide the starting point ([Dw3, p.83])", admitting that "it represents the goal toward which education should continuously move ([Dw3, p.83])", or that "the problem of teaching is to keep the experience of the student moving in the direction of what the expert already knows ([Dw1, p.184])". Indeed, in his view, "science is experience becoming rational ([Dw1, p.225])".

Problem method is, as is mentioned above, originally conceived in order to create an intellectual way of study that is found in "laboratories and institutes of research". In particular, in the course of study applied this method, the content of subject-matter is of essential meaning.

Thus, the problem-method considered by Dewey has nothing to do with the 'problem-solving' as an educational doctrine of today which is made an issue in the chapter 4. In the latter case, problems are no more than materials for the study of the concepts of those performances that are regarded to constitute the process of problem-solving; the content of subject-matter is, therefore, a matter of secondary importance.

As is pointed in the next section, it is difficult, if not impossible, to let the problem method to work in elementary schools. Dewey School was not exceptional on this point as far

as we admit that problem-solving was introduced there merely in order to instruct the concept of problem-solving, and to condition children as 'problem-solvers'. In fact, each child was habituated to think "before doing in all of his various enterprises", to be conscious of "direction of his actions toward considered social ends", and therefore to "postpone actions for longer and longer periods in order to perfect means to attain desired ends" ([MaE, pp.420–424]). Mayhew and Edwards stated:

"The ever-fresh activities of the school demanded a method of seeing and stating problems, of collecting facts, of acquiring materials and necessary skills, of planning the procedure of solution, and of executing the plans. While the problems of each day were new, the method of meeting them became a habit." ([MaE, pp.430, 431])

2-3. Points at issue about Dewey's methodology and about his estimation for problem method As is mentioned in §2-1, the experiment in Dewey School could be characterized as a part of the 'social inquiry'. Indeed, Dewey considered the education in the context of the 'growth', and for him the 'growth' is nothing but a reflection of the society, as is understood from the

following citation:

"It was held that the process of mental development is essentially a social process, a process of participation; traditional psychology was criticized on the ground that it treated the growth of mind as one which occurs in individuals in contact with a merely physical environment of things." ([MaE, p.467])

The concept of 'social inquiry'. however, holds on the certification of "Positivism". And positivism is, as is generally known, an old and new issue of criticism. For example, phenomenologists possibly criticize positivism on the ground that even positive science merely completes, through academic method, the knowledges that were already completed through prescientific cognition. That is, in positive science, those that were already established are premised.

Dewey aimed at establishing such education programs that are exactly based on the children's potential for facilitating further growth and learning. But it is impossible to consider any growth and learning with the content being left free. Growth and learning must be filled with concrete contents, and this is the very reason why 'growth' as a social process cannot be premised when programs of education are tried to establish. And here also happens a serious problem as was pointed by H. L. Friess as follows:

"Social inquiry must operate with some estimate of it, but the process and results of inquiry may change a situation's limits significantly." ([F, p.114])

This principle may be named the "uncertainty principle" following the famous principle in quantum mechanics.

Now let us make the efficacy of problem method an issue. Dewey stated:

"It is possible to find problems and projects that come within the scope and capacities of the experience of the learner and which have a sufficiently long span of that they raise new questions, introduce new and related undertaking, and create a demand for fresh knowledge." ([Dw2, pp.31, 32])

But the point is that the extent to which such problems span a life of learner necessarily depends on learner's intellectual level.

Research institutes are the very case where problem method completely works. In elementary schools, however, it is difficult to make problem method work. In fact, there a problem-solving generally seems to span only a very short unit of children's discrete experiences. Some reasons could be pointed, considering children's general characteristics. First of all, young children are in the process of obtaining social concepts and norms, and, as Gagné said, one cannot think "in a vacuum" ([Ga, p.175]). Such developmental level of intelligence necessarily decides a type of interest, and, finally, a form of instruction-learning process, where children's direct acceptance of the adult's culture appears to be of much positive significance. Thus, in order that programs can be constructed with any systematic problem-solving, a sufficient development of intellectual abilities must be premised.

We should mind that studies under problem method is enlivened generally in the case where a once acquired cognitive scheme is made a critical issue and one intends to dissolve it into confusion in order to reconstruct a satisfying scheme anew. But, in the case of a young child in a way to acquisition of social cognitive schemes, who therefore does not have sound schemes, the confusion caused by a failure in a new domain of cognition is necessarily so serious that he should lose his way to go back his already acquired schemes. Furthermore, young children do not have sufficient norms to evaluate their own schemes. Thus, as far as instructional efficacy concerns, contextual and integrated understanding of an object by means of problems should follow a sound identification, in any — but socially assured — form, of the object.

## III. Progressivists' view on education and problems approach

In the case of progressivism on education, the ultimate objective of education is decided to be preparing people to live effectively in a democratic society and to sustain the society, as participants in policy-making. This conception is based on a political stand opposing to monopoly and insisting the guard of democratic societies against monopoly. Progressivists thought that the system of monopoly is confirmed by authorized rigid dogmas which lasts as far as creative and critical way of thinking is suppressed and obedience is regarded as a good trait. They, therefore, emphasized creative, critical, reflective, rational and clear thinking with which one can resist authoritarianism as personal characteristics essential to democratic living and as the very intellectual ability that, through education, children in democratic societies should obtain. And as a necessary condition to bring up this ability they stressed on the freedom of the child to develop naturally and opposed to authoritarian controlment.

The problems approach as a method of education was considered by progressivists in this context, as was stated by S. P. McCutchen as follows:

"The problems approach ..... is an attempt to meet more directly and logically the demands democracy makes upon its devotees and upon the schools that train them." ([Mc, p.536]) Thus, every school subject was positioned as a part in the general education which was then characterized as means to bring up the disposition and ability to use reflective thinking in the analysis and solution of problem situations.

The Progressive Education Association (abreviated, PEA) made clear its stand respecting mathematics education in "Mathematics in General Education" published in 1940 ([P]). In the same year, the Joint Commission of the Mathematical Association of America and NCTM published a final report titled "The Place of Mathematics in Secondary Education" ([J]). As a background of the two reports there was a decline of prestige of mathematics in school subjects, which was essentially caused by the uprising of the percentage of school attendance and by resulting multiformity of students' interests and needs, and, accordingly, there was a sense of crisis growing among mathematics educationists.

The PEA Report, deciding the purpose of general education as the provision of "rich and significant experiences in the major aspects of living, so directed as to promote the fullest possible realization of personal potentialities, and the most effective participation in a democratic society ([P, p.43])", considered "the role of mathematics in achieving the purpose of general education".

In the report, it is stated that:

"The development of intelligence in analyzing problem situations, otherwise referred to as reflective thinking, although but a part of the purpose of general education, is so essential a part as to be given a major place in this Report." ([P, p.52]).

Then the development of reflective thinking, or problem-solving, was decided as the major role of mathematics education. The report stated:

"....the study of mathematics is of educational value because mathematics can be made to throw the problem-solving process into sharp relief, and so offers opportunity to improve student's thinking in all fields." ([P, p.60])

It is noteworthy that in the report the teacher's task for the development of children's problem-solving-ability is specified to be making the concepts of "formulation and solution", "data", "approximation", "function", "operation", "proof", and "symbolism" be understood. Thus, the "problem-solving-ability" considered there is actually nothing but a knowledge about these concepts or a habit, or a mental set, to use the concepts as repertories of problem-solving schemes. And, therefore, the concept of "problem-solving-ability" turns out to be content- and context-free. This fact should be considered in contradistinction to the conception of "problem-solving" in the context of problem method which was regarded by Dewey as an alternative to the then traditional method of education.

Progressivists stressed on "immediate experience" and "individual concerns (interests and felt needs)". W. H. Kilpatrick, specifying his learning theory, decided that "each one learns what he lives", that is, "he learns his responses, only his responses and all his responses" and, therefore, that "he learns each such accepted response in the degree that he counts it important and in the degree that it interrelated itself with what he already knows" ([KIW, p.483]). Then he concluded as an educational implication of the theory that "the school should be primarily a place for living", living all aspects of life, and that "if we wish him to learn anything ....., he must first live that thing" ([KIW, p.483]).

Here are spotlighted two aspects of subject-matter, that is, subject-matter as material for the learning of modes of clear thinking and problem-solving and one that represents some aspects of life. They must be compatible as long as progressivism goes coherently. The most difficult, however, is not to realized this compatibility and bring the realities of life into the class (not questioning whether so-called progressive schools actually realized this ideal), but to assure certain intellectual level of members of the society by instructing such subject-matters.

Progressivists critisized the content-mastery instruction, judging that it stood delayed-effect rationalization. In fact, they regarded, on the ground of relativism, the anticipation of future needs as a gambling. But, needless to say, there is naturally a limit to such "gambling". Furthermore, we must not forget a major function of education, that is, preparing people to sustain and draw up the developmental level of the society. And in this case it is the content of subject-matter that should be considered to be of the first importance.

It should be noted that incidental learning following the doctorine of immediate interests or concerns could not constitute a school subject, as was pointed by W. Betz as follows:

"It has been found impossible, even in the field of arithmetic, to arrange life situations sequentially in such a way that mathematical concepts, principles and processes can be built up, with their aid, in the cumulative manner which is essential in mathematics." ([B, p.352])

Indeed, "life situations" can merely endorse parts of mathematics discretely. Thus, it could be regarded as a natural consequence that so-called progressive schools failed to realize "continuity of educational experience" and, in fact, treated "pupils as a mere succession of cross-sections" ([MaE, p.469])

## IV. Conception of 'problem-solving' of today

The doctrine of mathematical 'problem-solving' of today decides the upbringing of so-called "good problem-solvers", or "good thinkers", to be an ultimate objective or, at least, one of the most important objectives of mathematics education. The upbringing of "good problem-solvers" is practically the same as what is advocated in the aforementioned PEA's report as one of the major objectives of general as well as mathematics education, that is, the development of "reflective thinking". In fact, judging from the fact that advocates of 'problem-solving' in the United States generally insist on the unknowableness of problems in the future, we may also suppose that the idea of 'problem-solving' of today, too, stands on the relativism respecting subject-matters. Thus, when it is seen on the level of philosophy, there is nothing new in the doctrine as far as it is compared with the corresponding conception in progressivism.

The characteristic of the current conception of 'problem-solving', however, is acknowledged in the respect that the theorists intend to introduce and apply methods and concepts which psychologists use in their studies on "human problem-solving" (see, for example, [L, p. 13], [KIJ1, p.523]). Precisely, psychologists consider problem-solving as "basic cognitive process" (cf. [Da], [Ga, Ch.7], [Gr]), and, following them, the theorists of mathematical 'problem-solving' of today consider the instruction-learning process of problem-solving as a psychological cognitive process and in terms of the acquisition of "problem-solving-strategies" (e. g. [KIJ2]).

Problem-solving as a performance is regarded by the theorists in the United States as a manifestation of "skills for problem-solving". Thus, one's inability for problem-solving is judged to be a consequence of the fact that some of needed "skills" are lacking. And "strategies for problem-solving" that the theorists conceive are nothing but the products of their schematization of those "skills". Thus, since the inability for problem-solving is understood by such causation, the remedy for it turns out to be the *prosthesis* where "skills" considered to be

lacking is directly filled up by the instruction of corresponding "strategies" for problemsolving.

Since, as is mentioned above, the conception of 'problem-solving' stands on a sort of relativism respecting subject-matter, the so-called "strategies for problem-solving" necessarily turn out to be content- and context-free. In fact, what is considered by the theorists of mathematical 'problem-solving' as "stratigies for problem-solving" are such as "to understand the problem", "to identify the unknown", "to make a plan for the solution", "to collect and rate data", "to simplify the situation" and "to devise a graphic representation" (see, for example, [C]). Such are the very strategies they consider to be effective when one copes with "real-world problems".

But here happens a question. Are the 'competences' expressed above in the form of "strategies" exist as real competences? For example, can one consider "to make a plan for the solution" as a definite competence? On the contrary, from the author's view, it is a mere concept meaning a category of performances called equally "to make a plan for the solution". Needless to say, to know a category of performances does not assure to be able to realize each performance in the category. These concepts are so broad that the possible and most significant function of the instruction in them is necessarily to make students obtain a habit or a mental set to face a problem in such a definite manner that is specified by the concepts. What is forgotten or, at least, seriously neglected by the advocates of mathematical 'problem-solving' in the United States is the *next* stage, that is, to make one obtain such competences that enables him to wrestle substantially with concrete problems.

## V. Objectives of mathematics education

### 5-1. Self-realization

The Educational Policies Commission of the National Education Association in the United States identified in the report on "The Purposes of Education in American Democracy" (1938) the 'self-realization' as one of major purposes of education. In fact, the commission advocated four purposes, that is, "self-realization", "human relationship", "economic efficiency" and "civic responsibility" ( [E1], [E2], or cf. [Kn]). From the author's view, however, the latter three objectives could be subordinated to the first one; the objective of self-realization. Indeed, understanding 'self-realization' in an adequate broad sense, we could say even that education means the spurring letting one to complete some kind of "meaningful" (which is, of course, a concept depending on situations) 'self-realization'

The variation of functions of education corresponds to the various possible ways of self-

realization. The meaning of existence of school mathematics should be considered in this context. That is, what must be made an issue is the possible results of "studying school mathematics" which contribute toward some modes of self-realization. Mathematics education is significant also in the sense that it functions as a moment which evokes some meaningful 'self-realization' in students.

The modes of significance of mathematics education, therefore, turn out to depend on the given situation and on the aptitudes of individuals. Utilities are not inherent in mathematics itself. They are not objects of educational study untill they are considered in the context of 'self-realization'. Indeed, utilities can be discussed only in such form as 'in the situation one could self-realize as ..... [e. g., a citizen, a mathematician, an economic man, a thinker] through studying mathematics".

Thus, to be a "good problem-solver" which is usually worded to mean one who masters the so-called "modes of thinking and problem-solving" and is able to apply them, is only one of various possible forms of 'self-realization'. And, therefore, education should necessarily become distorted if its general objective is decided to be the upbringing of "problem-solvers". On the other hand, in a situation like ours where the meaning of existence and the values are multiplicate, one cannot set out *general* objectives of education without making education vague and obscure. "For everybody" is actually "for nobody".

Various types of 'self-realization' can be pointed as results of learning of mathematics. Thus, it is necessary that the multiplicity of the meaning of mathematics education is conceptualized. This conception induces several problems. For example, establishment of orientation system that sufficiently corresponds to the variety of students' trait, and introduction of a carefully thought out elective system under which subject-matter used in each course is clearly characterized following some specified objective.

Now let us consider 'heuristics' as a topic concerning 'self-realization'. It is originally conceptualized as a way of instruction to make students realize themselves as (scholatic) inquirers. It could, thus, be defined as an instructional methodology which aims at evoking students' self-realization as seemingly "authoritative" inquirers by letting them to become concious of their potentiality to do a thing of social value or to assimilate an act of an 'authority'. The most essential to the methodology is to make one become concious of his own potentiality, not only to release it. Needless to say, such 'self-realization' is not a final end. Being linked with 'confidence' and 'motivation', it is regarded as a moment of further qualified and higher-ordered studies.

Heuristics relates to the doctrine of 'problem-solving' in such context as "heuristics for

the purpose of evoking self-realization as a good problem-solver or a good thinker". Here the way of the instruction, as a matter of course, depends how 'problem-solver' is defined.

## 5-2. Understanding of meta-world

Upbringing the ability of "creative thinking" is generally identified as one of the major objectives of education. This objective is not subsumed in 'problem-solving' because every new way of thinking founds, in greater or lesser degree, on existing way of thinking a part of which is substantialized as school subject-matter and, therefore, because well-oriented subject-mastery is needed to achieve the objective.

The wording of "creative thinking" is somewhat misleading. To be concrete, the process of creative thinking is the introduction of a meta-conception where a given problem situation appears to be adequately located and soluted. Creative thinking is nothing but creating and understanding of a meta-world.

Thus, on the extension line of subject-learning, that is, of learning of the world, there exists learning of the methodology for understanding the world, that is, learning of meta-world. Education for the purpose of pushing up the developmental level of society must necessarily include such aspect of learning.

### 5-3. Learning of concepts of "clear thinking" and "problem-solving"

The conception of "clear thinking" and "problem-solving" as we see is definitely a product of a peculiar culture, and thus, it is of relative significance. It is such object that must be intentionally instructed. Precisely, one must learn what problem or problem-solving means, what the process of problem-solving is, what it means to see materials with a critical mind, what it means to find those that are "essential", and so on.

As is already mentioned, the learning of such concepts is the very thing that the advocates of 'problem-solving' in the United States decide to be the direct objective of the problems approach. They aim at making students to act as 'clear thinkers' or as 'problem-solvers' and to study by thinking, not by memorizing.

The acquisition of general concepts about "clear thinking" and "problem-solving" is, however, merely a necessary, but not sufficient, condition for each specific concrete problem being successfully solved. To understand the concepts is one thing and to treat actual world problems is another. A practical problem-solver needs many concrete models of problem-solving on the analogy of which he can solve problems and, of course, ability to learn anew. And to have one obtain them is out of range that 'problem-solving' instruction can cover. In

this sense, it is necessary for pedagogists to make clear the limit of 'problem-solving' instruction.

#### References

- [B] Betz, W. The present situation in secondary mathematics, with particular reference to the new national reports on the place of mathematics in education. Math. Teacher 33 (1940), pp.339-360.
- [C] Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Jr., Lindquist, M. M., & Reys, R. E. NAEP Note: Problem solving. Math. Teacher 73 (1980), pp.427-433.
- [Da] Davis, G. A. Current status of research and theory in human problem solving. Psychol. Bull. 66 (1966), pp.36-54.
- [Dw1] Dewey, J. Democracy and education —— An introduction to the philosophy of education. The Free Press, 1966. (Reprint of the 1916 ed. published by Macmillan Company)
- [Dw2] . The way out of educational confusion. Harvard Univ. Press, 1931.
- [Dw3] ———. Experience and education. Collier Books, 1963. (Reprint of the 1938 ed. published by Kappa Delta Pi)
- [E1] The Educational Policies Commission of the National Education Association in the United States.

  The purposes of education in American democracy (an excerpt). School & Society 48 (1938), p.620.
- [E2] ——. The purposes of education in American democracy (an excerpt). Educ. Digest 4 (1930), pp.1-4.
- [F] Friess, H. L. Social inquiry and social doctrine. In Hook, S. (ed.), John Dewey: Philosopher of science and freedom. Dial Press, 1950, pp.106-117.
- [Ga] Gagné, R. M. The conditions of learning (3rd ed.). Holt, Rinehart and Winston, 1977.
- [Gr] Green, B. F., Jr. Current trends in problem solving. In Kleinmuntz, B. (ed.), Problem solving: Research, method and theory. New York: Wiley & Sons, 1966, pp.3-18.
- [J] The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics. The place of mathematics in secondary education —— The final report of the Joint Commission of MAA and NCTM. 15th Yearbook of NCTM, 1940.
- [KIJ<sub>1</sub>] Kilpatrick, J. Problem solving in mathematics. Rev. Educ. Research 39 (1969), pp.523-534.
- [KIJ2] ———. Variables and methodologies in research on problem solving. In Hatfield, L. L. (ed.), Mathematical problem solving, Columbus, Ohio: Eric Clearinghouse for Science, Mathematics, and Environmental Education, 1978, pp.7-20.
- [KIW] Kilpatrick, W. H. The philosophy of the New Education. School & Society 54 (1941), pp.481-484.
- [Kn] Kinney, L. B. Why teach mathematics? Math. Teacher 35 (1942), pp.169-174.
- [L] Lester, F. K., Jr. Idears about problem solving: A look at some psychological research. Arithmetic Teacher 25 (1977, Nov.), pp.12-14.
- [MaE] Mayhew, K. C., & Edwards, A. C. The Dewey School. Atherton Press, 1966. (Reprint of the 1936 ed. published by D. Appleton-Century Co. Inc.)

- [Mc] McCutchen, S. P. The problems approach to the social studies. J. Educ. Sociology 20 (1947), pp.529-536.
- [N] The National Council of Teachers of Mathematics. An agenda for action —— Recommendations for school mathematics of the 1980s. NCTM, Inc., 1980.
- [P] The Progressive Education Association. Mathematics in general education: A report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum. New York: Appleton-Century-Crofts, Inc., 1940.